Distribution Network Planning for Franchised Restaurants under Uncertain Demands

Project Report of Stochastic Optimization Course

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***Abstract*—A two stage stochastic programming model is built to design the distribution network for a Cantonese franchised restaurants, answering distribution centers' order quantity of each supplier. Extended Pearson-Tuck (EPT) approximation of continues distribution and Monte Carlo simulation are used to solve the model.**

Keywords-distribution network; stochastic optimization; two stage; EPT; Monte Carlo Simulation.

I. Introduction

A Cantonese franchised restaurant company is now growing fast in southeast of China. It wants to redesign its ingredients distribution network. The contract between the master company and its franchisees specifies that all the franchisees must use the processed ingredients from the master company. The master company has some potential raw ingredients suppliers. It also own some facilities, which is a combination of ingredients processing factory and distribution center. During each operation cycle, the distribution center give orders to the raw ingredients suppliers, have the raw ingredients transferred. Then the master company will receive demands orders from its franchisees, and asks the distribution centers to deliver certain amounts of processed ingredients to each franchisee. As the ingredients is easy to perish, the processed ingredients will be in storage for just the current operation cycle.

The company now has franchisees located at 33 different cities, and has constructed distribution terminal at each city, which is responsible for summarize a city's demand order and the final delivery. Some analyze had been done and shows that the summarized demand of each city subject to independent normal distributions. Now the operation team of the distribution center try to plan a new network that will fulfill the uncertain and minimize the cost.

II. Data Sets

A. Data Sets Related to the Quantity of Ingredients

There are 27 potential raw material suppliers. Each pf them has a capacity limitation of how much raw ingredients then can provide, in unit of ton. The contract between the master company and the suppliers agreed that the distribution center should give the same order amount for every operation cycle. So there is no uncertainty here.

There are summarized demand of 33 different cities, each of them subject to a normal distribution, with estimated mean and estimated standard deviation, in the unit of ton. The contract gives flexibility to the franchisees' demand order. And the distribution center can fail to fulfill some demands with some penalty cost, in the unit of \$/ton. The penalty is 1.35 in this case.

B. Data Sets Related to Transportation

The distribution center is responsible for the cost of highway transportation of all kinds of ingredients, which is in the unit of \$/(km×ton). The cost is 0.0012 in this case.

The 27 suppliers, 3 distribution centers and the 33 terminal cities can all be abstracted as points on a map. There is a distance matrix that contains the highway distance in km between each two points. The matrix can be separated into two simplified matrix, which are supplier-DC matrix and DC-city matrix.

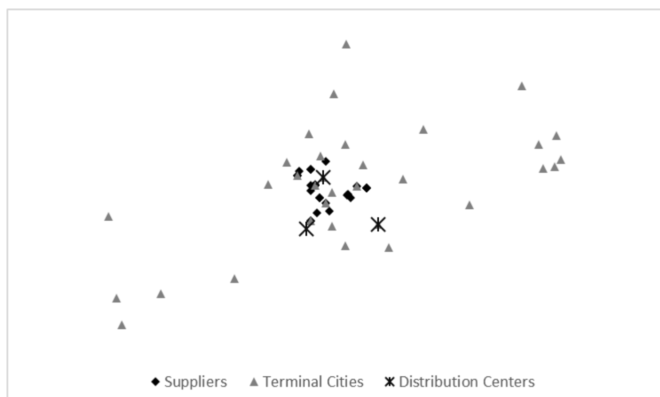

Figure 1. Locations of Suppliers, Terminal Cities & Distribution Centers

III. Two Stage Stochastic Programming Model

It is naturally to model this problem as a two stage stochastic programming. At the first stage, the operation team of distribution centers needs to decide their order quantity of each supplier to each distribution center, without knowing the future realization of the demand order of each city. At the second stage, the operation team of distribution centers need to determine the deliver amount from each distribution center to each city

A. Sets

S	set of 27 potential suppliers;
C	set of 3 distribution centers;
T	set of 33 terminal cities;

B. Inputs and parameters

CL_s	Capacity limit of supplier *s*, for each $s \in S$;
$\tilde{d}_t$	The uncertain demand of terminal t, for each $t \in T$;
$DS_{s,c}$	The distance between supplier *s* and distribution center *c*, for each $s \in S, c \in C$;
$DT_{t,c}$	The distance between terminal *t* and distribution center *c*, for each $t \in T, c \in C$;
q	The transportation cost for unit mass of ingredients along unit distance;
p	The penalty cost for unit mass unfulfilled demand;

C. Decision Variables

$x_{s,c}$	Quantity of raw ingredients transported from supplier *s* to distribution center *c*, for each $s \in S, c \in C$;
$y_{t,c}$	Quantity of processed ingredients that terminal *t* got delivered from distribution center *c*, for each $t \in T, c \in C$;
z_t	Quantity of unfulfilled demand of terminal *t*, for each $t \in T$;

D. Stochastic Model

Min $\quad \sum_{s\in S}\sum_{c\in C}\left(q \cdot DS_{s,c}x_{s,c}\right) + Eh\left(x,\tilde{d}\right)$

s.t. $\quad \sum_{c\in C} x_{s,c} \le CL_s \quad$ for each $s \in S$ (1)

$x_{s,c} \ge 0 \quad$ for each $s \in S, c \in C$ (2)

Where $h\left(x,\tilde{d}\right)$ is

Min $\quad \sum_{t\in T}\sum_{c\in C}\left(q \cdot DT_{t,c}y_{s,c}\right) + \sum_{t\in T} p \cdot z_t$

s.t. $\quad \sum_{t\in T} y_{t,c} \le \sum_{s\in S} x_{s,c} \quad$ for each $c \in C$ (3)

$\sum_{c\in C} y_{t,c} + z_t \ge \tilde{d}_t \quad$ for each $t \in T$ (4)

$y_{t,c} \ge 0 \quad$ for each $t \in T, c \in C$ (5)

$z_t \ge 0 \quad$ for each $t \in T$ (6)

Constraint (1) says that for each supplier, the sum of order amount from all distribution center of a supplier can't exceed its capacity. Constraint (3) says that for each distribution center, the quantity of ingredient it delivered should not exceed the quantity of ingredient it received. Constraint (4) says that for each terminal city, the quantity of ingredient delivered from all the distribution center plus the unfulfilled demand should more than its demand order.

The two objective function can be combined together as

Minimize $\sum_{s\in S}\sum_{c\in C}\left(q \cdot DS_{s,c}x_{s,c}\right) + \sum_{t\in T}\sum_{c\in C}\left(q \cdot DT_{t,c}y_{s,c}\right) + \sum_{t\in T} p \cdot z_t$

It contains three parts. The transportation cost of get raw ingredient from suppliers to distribution centers. The transportation cost of deliver processed ingredient from distribution centers to terminal cities. And the penalty cost for unfulfilled demand orders. Combine the new objective function and constraints (1)-(5), we can get the simplified stochastic model. Since we know that each $\tilde{d}_t$ subject to a normal distribution and we have the estimated mean and standard deviation, we can solve the problem Monte Carlo Simulation method.

IV. Two Stage Model with Discrete Scenario

Instead of using Monte Carlo Simulation method to get a statistical result from solving this model for several times, we first introduce the way of building a scenario model that is deterministic and linear, and we can solve it to get an exact solution. The key issue here is that it is necessary to have discrete distribution for random variables to construct scenario, while we just have parameters of the continues normal distribution.

There are several methods that have been promoted to do the discrete approximation of continues distribution. Some researchers showed that the Extended Pearson-Tuck (EPT) method works well on the discrete approximation of normal distribution. The EPT use the 5%, 50% and 95% quantile and give them the probability of 0.185, 0.63 and 0.185. Here the EPT was used to generate a 33×3 discrete distribution matrix and the 33×3 probability matrix. We can generate the scenario matrix from the discrete distribution matrix, by choosing one value from each row and from a new column. The probability of each scenario (row) is the production of the corresponded probability of each row's element.

It is obvious that there are too many (3^33) scenarios, which will lead to too many constraints. A procedure which will be discussed later is take to reform the terminal and make the number of terminal decrease to 8, thus the number of scenario is decreased significantly. Then the two stage model with scenario can be modified from the original one.

A. Sets

S	set of 27 potential suppliers;
C	set of 3 distribution centers;
T	set of 33 terminal cities;
Ω	set of scenarios.

B. Inputs and parameters

CL_s	Capacity limit of supplier *s*, for each $s \in S$;
d_t^{ω}	The demand of terminal t under scenario ω, for each $t \in T, \omega \in \Omega$;
r^{ω}	The probability of scenario ω, for each $\omega \in \Omega$;
$DS_{s,c}$	The distance between supplier *s* and distribution center *c*, for each $s \in S, c \in C$;
$DT_{t,c}$	The distance between terminal *t* and distribution center *c*, for each $t \in T, c \in C$;
q	The transportation cost for unit mass of ingredients along unit distance;
p	The penalty cost for unit mass unfulfilled demand;

C. Decision Variables

$x_{s,c}$	Quantity of raw ingredients transported from supplier *s* to distribution center *c*, for each $s \in S, c \in C$;

$y_{t,c}^{\omega}$ Quantity of processed ingredients that terminal t got delivered from distribution center c under scenario ω, for each $t \in T, c \in C,\ \omega \in \Omega$;

z_t^{ω} Quantity of unfulfilled demand of terminal t under scenario ω, for each $t \in T,\ \omega \in \Omega$;

D. Scenario Model

Min
$$\sum_{s\in S}\sum_{c\in C}\left(q \cdot DS_{s,c}x_{s,c}\right) + \sum_{\omega\in\Omega} r^{\omega}\left[\sum_{t\in T}\sum_{c\in C}\left(q \cdot DT_{t,c}y_{t,c}^{\omega}\right) + \sum_{t\in T} p \cdot z_t^{\omega}\right]$$

s.t.
$$\sum_{c\in C} x_{s,c} \le CL_s \quad \text{for each } s \in S$$
$$x_{s,c} \ge 0 \quad \text{for each } s \in S, c \in C$$
$$\sum_{t\in T} y_{t,c}^{\omega} \le \sum_{s\in S} x_{s,c} \quad \text{for each } c \in C,\ \omega \in \Omega$$
$$\sum_{c\in C} y_{t,c}^{\omega} + z_t^{\omega} \ge d_t^{\omega} \quad \text{for each } t \in T,\ \omega \in \Omega$$
$$y_{t,c}^{\omega} \ge 0 \quad \text{for each } t \in T, c \in C,\ \omega \in \Omega$$
$$z_t^{\omega} \ge 0 \quad \text{for each } t \in T,\ \omega \in \Omega$$

V. Simplification of Scenario Model

As mentioned above, the large number of scenario make it almost impossible to implement the scenario model and get it solved. To get an approximation that can reduce the scenario, we separate the 33 terminals into some number of clans, and one city in each clan to be a representative. Imagine that the representative just work as a virtual low level distribution center, that the demand order of every terminal in this clan come together to the representative and then the representative report to the real distribution center.

We would like to have as much number of representatives as possible, since more representatives is closer to the real situation. Meanwhile too much representatives will make it meaningless to reduce the number of scenario. The number is set to no more than 8 to get a balance finally.

A location allocation model is built to find the separation of clans and their representatives. The objection is to minimize the production of total quantity of ingredients and the transported distance. To simplify this procedure, the mean demand is used as an approximation of demand order.

A. Sets

T set of 33 terminal cities;

C set of 3 distribution centers;

B. Inputs and parameters

d_i The demand of terminal i, for each $i \in T$;

$DT_{c,j}$ The distance between distribution center c and terminal t, for each $c \in C, j \in T$;

$DTT_{i,j}$ The distance between terminal i and terminal j, for each $i \in T, j \in T$;

C. Decision Variables

x_j Indicator variable of whether terminal j is selected to be a representative, for each $j \in T$;

$y_{i,j}$ Indicator variable of terminal t is allocated to the representative terminal j, for each $i \in T, j \in T$;

$z_{c,j}$ Quantity of ingredients that distribution center c send to terminal j, for each $c \in C, j \in T$

D. Allocation Model

Min
$$\sum_{j\in T}\sum_{i\in T}\left(d_i DTT_{i,j}y_{i,j}\right) + \sum_{j\in T}\sum_{c\in C} z_{c,j}DT_{c,j}$$

s.t.
$$\sum_{j\in T} y_{i,j} = 1 \quad \text{for each } i \in T$$
$$\sum_{c\in C} z_{c,j} = \sum_{i\in T} d_i y_{i,j} \quad \text{for each } c \in C, j \in T$$
$$y_{i,j} \le x_j \quad \text{for each } i \in T,\ \ j \in T$$
$$\sum_{i\in T} x_j \le 8 \quad \text{for each } j \in T$$
$$x_j \in \{0,1\} \quad \text{for each } i \in T$$
$$y_{i,j} \in \{0,1\} \quad \text{for each } i \in T, j \in T$$

The objective function has two parts. The first part is about the ingredients transported from terminals to the potential representatives. The second part has the ingredient that transported from distribution centers to representatives considered.

E. Allocation Results

8 clans and their representatives have been found.

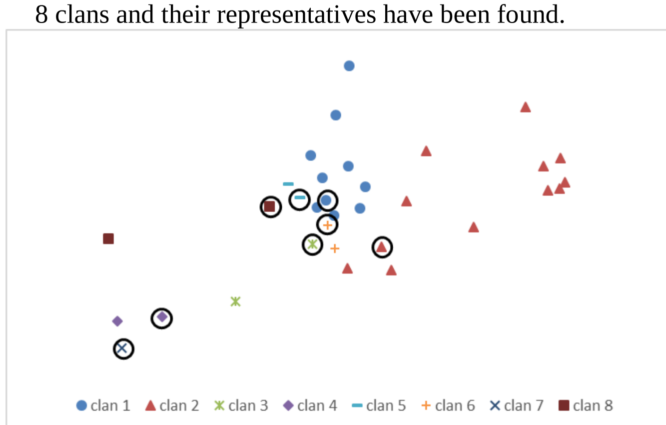

Figure 2. Allocation and Representatives of Terminals

The results shows that clan 1 are connected to distribution center 1, clan 2 are connected to distribution center 2, and all the other clans are connected to distribution center 3.

The virtual demands of the representatives are regard as a normal distribution, which is the sum of normal distributions of its clan member's demand. Thus the EPT is applied to these 8 representatives to build scenarios. The number of scenario is reduced to 3^8.

VI. Two Stage Model with Monte Carlo Simulation

The Monte Carlo Method used here can be regarded as a statistical way to generate scenarios. In each loop we generate a realization of the uncertainty according to the distribution of the random variables, then add the constraints and its part of objective function correspond to this scenario to the model. Each scenario has a weight of 1/n.

It has shown that the Discrete Scenario may add too many constraints. The Monte Carlo Method may find an approximate optimal solution with much less constraints.

VII. Implement and Solve the Models

All the models are implemented by YALMIP, a MATLAB toolbox used for mathematical programing modeling. YALMIP has the ability to connect to several mainly used solvers. In this case CPLEX is selected to be the solver.

The results firstly focus on the expected cost, which is the value of objective function. The network between supplier and distribution center is also emphasized, which is the value of decision variable $x_{s,c}$. There is not a general solution of the network between distribution centers and terminal cities, this network will be determined as soon as the distribution center receive the demand order.

A. *Solve the Two Stage Model with Discrete Scenario*

The objection value is 705,913. It should be expected that the objection value is less than the real expected total cost under this supplier network since the clan allocation procedure has made several transportation be eliminated in the final model. The supplier network, is shown in Table 1.

Table 1. Optimal Supplier Network with Discrete Scenario Model

	Capacity	DC1	DC2	DC3
Supplier1	350000	0	0	350000
Supplier2	450000	0	0	450000
Supplier3	350000	0	0	0
Supplier4	350000	350000	0	0
Supplier5	350000	350000	0	0
Supplier6	350000	0	350000	0
Supplier7	350000	0	350000	0
Supplier8	450000	94296	96053	0
Supplier9	450000	0	450000	0
Supplier10	350000	0	0	0
Supplier11	350000	0	0	350000
Supplier12	350000	350000	0	0
Supplier13	350000	0	0	350000
Supplier14	350000	0	0	350000
Supplier15	350000	0	350000	0
Supplier16	350000	0	350000	0
Supplier17	350000	350000	0	0
Supplier18	350000	0	0	0
Supplier19	350000	0	0	350000
Supplier20	350000	0	0	0
Supplier21	350000	0	0	350000
Supplier22	450000	450000	0	0
Supplier23	350000	0	350000	0
Supplier24	350000	0	0	0
Supplier25	350000	0	350000	0
Supplier26	350000	0	0	350000
Supplier27	350000	0	0	350000
Sum		2692513	24720	5000000

We can see that the total amount the DC orders is 7,717,233, locating between the total demands of mean scenario (6,476,746) and the worst scenario (9,020,422). It means the model help the operation team find a balance between the risk of penalty of unfulfilled demand and over processed ingredients.

B. *Solve the two stage model with Monte Carlo Method*

The objection value is 731,746, with a 95% confidence interval of [702,476, 761,016]. The supplier network is shown in table 2.

Table 2. Optimal Supplier Network with Monte Carlo Method

	Capacity	DC1	DC2	DC3
Supplier1	350000	0	0	350000
Supplier2	450000	0	0	450000
Supplier3	350000	0	0	0
Supplier4	350000	350000	0	0
Supplier5	350000	350000	0	0
Supplier6	350000	0	350000	0
Supplier7	350000	0	350000	0
Supplier8	450000	183035	0	0
Supplier9	450000	0	450000	0
Supplier10	350000	0	0	0
Supplier11	350000	0	0	350000
Supplier12	350000	350000	0	0
Supplier13	350000	0	0	350000
Supplier14	350000	0	0	350000
Supplier15	350000	0	350000	0
Supplier16	350000	0	350000	0
Supplier17	350000	350000	0	0
Supplier18	350000	0	0	23538
Supplier19	350000	0	0	350000
Supplier20	350000	0	0	0
Supplier21	350000	0	0	350000
Supplier22	450000	450000	0	0
Supplier23	350000	0	350000	0
Supplier24	350000	0	0	0
Supplier25	350000	0	350000	0
Supplier26	350000	0	0	350000
Supplier27	350000	0	0	350000
Sum		737682	3548702	3250000

We can see that the total amount of the DC orders is 7,856,573, which is very close, but a little higher than the result of the solution with discrete scenario..

C. *Results Compare*

We can see that the results of supplier network are almost the same, except for supplier 8 and supplier 18. A simulation experiment is designed to see that given the supplier network, what the operation cost is under uncertain demand.

According to the experiment, the expected cost under the supplier network of discrete scenario method is about 737,000, which is a little larger than its result. The expected cost under the supplier network of Monte Carlo method is about 733,000, which is very close to its result. The supplier network of Monte Carlo method just performs a little better than that of Discrete

Scenario Model. It reveals the fact that EPT discretion actually shares the same insight with Monte Carlo Simulation.

As a comparison, the experiment result of the simple ES solution (we use the mean value instead of the random variable) is about 1,001,000. That is worse than what we get before.

VIII. CONCLUSION

The distribution network planning problem is solved according to the two stage stochastic programming model. When face large amount uncertainty subjects to continuous distribution, the Monte Carlo Simulation is a good approach to solve the problem with less constraints in the model and less solving time to use. If the concept of simulation is not preferred, we can also use an appropriate discretion to approximate the continuous random variable. With proper ways to aggregate the random variables together, we can significantly reduce the number of uncertainties and build a two stage stochastic programming model that is solvable. And the result can still have good performance.

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